CSMC Practice #1)

2. An arithmetic sequence is a sequence in which each term after the first is obtained from the previous term by adding a constant d, called the common difference. For example, the sequence 2, 11, 20, 29, 38 is an arithmetic sequence with five terms and a common difference of $d = 9$.

   (a) An arithmetic sequence has three terms. The three terms add to 180. Determine the middle term of this sequence.

   (b) An arithmetic sequence has five terms. The five terms add to 180. Show that at least one of the five terms equals 36.

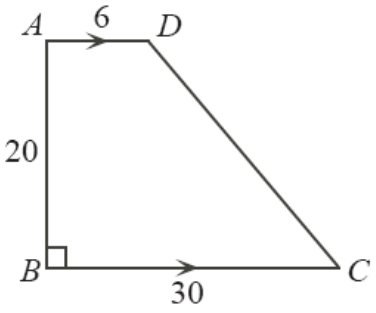
   (c) An arithmetic sequence has six terms. The six terms in the sequence add to 180. Determine the sum of the first and sixth terms of the sequence.

3. In the diagram, $ABCD$ is a trapezoid with AD parallel to BC and BC perpendicular to AB. Also, $AD = 6$, $AB = 20$, and $BC = 30$.

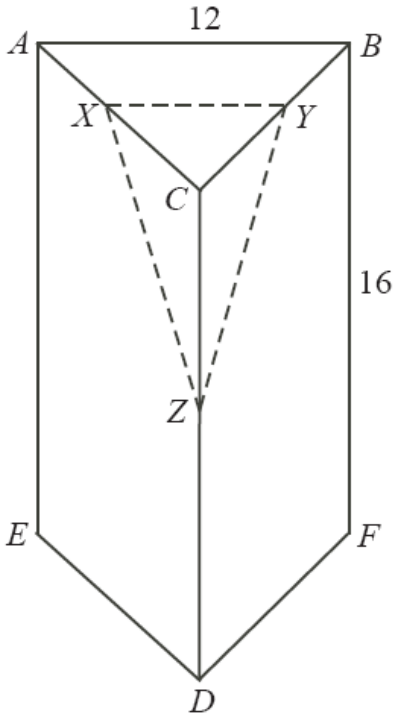
   (a) Determine the area of trapezoid $ABCD$.

   (b) There is a point K on AB such that the area of $\triangle KBC$ equals the area of quadrilateral $KADC$. Determine the length of BK.

   (c) There is a point M on DC such that the area of $\triangle MBC$ equals the area of quadrilateral $MBAD$. Determine the length of MC.

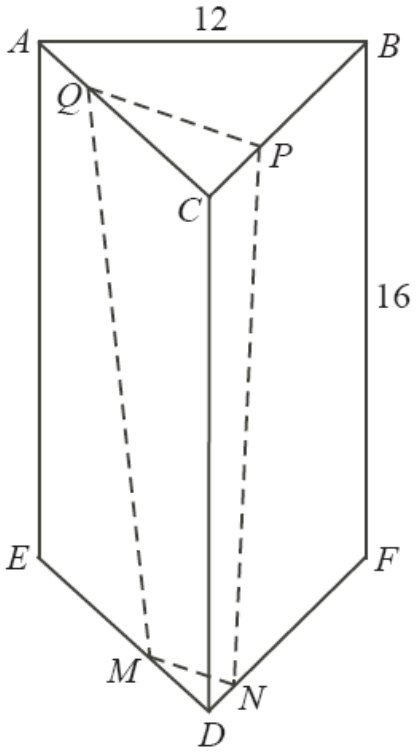


4. (a) A solid right prism $ABCDEF$ has a height of 16, as shown. Also, its bases are equilateral triangles with side length 12. Points X, Y, and Z are the midpoints of edges AC, BC, and DC, respectively. Determine the lengths of XY, YZ and XZ.



(b) A part of the prism above is sliced off with a straight cut through points X, Y and Z. Determine the surface area of solid $CXYZ$, the part that was sliced off.

(c) The prism $ABCDEF$ in part (a) is sliced with a straight cut through points M, N, P, and Q on edges DE, DF, CB, and CA, respectively. If $DM = 4$, $DN = 2$, and $CQ = 8$, determine the volume of the solid $QPCDMN$.



4. The *peizi-sum* of a sequence $a_1, a_2, a_3, \ldots, a_n$ is formed by adding the products of all of the pairs of distinct terms in the sequence. For example, the peizi-sum of the sequence a_1, a_2, a_3, a_4 is $a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4$.

   (a) The peizi-sum of the sequence $2, 3, x, 2x$ is -7. Determine the possible values of x.

   (b) A sequence has 100 terms. Of these terms, m are equal to 1 and n are equal to -1. The rest of the terms are equal to 2. Determine, in terms of m and n, the number of pairs of distinct terms that have a product of 1.

   (c) A sequence has 100 terms, with each term equal to either 2 or -1. Determine, with justification, the minimum possible peizi-sum of the sequence.